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Development of Didactical Design in Volume Teaching: Efforts to Improve Critical Thinking Abilities

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Abstract

This study aims to identify and analyze the learning obstacles experienced by students in volume instruction in grade 5 elementary school, as well as to design didactical interventions that can enhance students' critical thinking skills. The study uses the Didactical Design Research (DDR) approach, focusing on the first stage, which is analyzing the didactical situation before teaching. The findings show a complex interaction between three types of learning obstacles: ontogenic, didactic, and epistemological. Ontogenic obstacles are related to individual students' cognitive development, didactic obstacles are linked to the teaching methods used, and epistemological obstacles arise when students struggle to understand abstract concepts. The study found that didactic obstacles often exacerbate ontogenic and epistemological obstacles. Based on these findings, a didactical design was developed using the framework of Didactical Situations Theory (TDS), which aims to create learning situations that allow students to build understanding through interaction with challenging yet accessible materials. Future research is expected to proceed to the stages of meta-pedagogical and retrospective analysis to evaluate the effectiveness of the developed didactical design in real-world learning contexts.

Keywords: Didactical design; Critical thinking; Learning obstacle

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1. Introduction

Critical thinking is an essential skill every individual requires in daily activities (Liu & Pásztor, 2022). Studies have shown that individuals with critical thinking skills can complete tasks effectively (Astawa et al., 2022; Wagner, 2022). Critical thinking is not only important for individuals but also necessary on a macro scale. In the life of a nation's society, a country's system can only develop if the people critically engage with political, economic, and social issues (Demir et al., 2023; Trilling & Fadel, 2009). Critical thinking has become an important discourse in the field of education and a primary learning goal in many countries, including Turkey, Malaysia, Singapore, and Indonesia (Akinoglu & Baykin, 2015; Baki et al., 2016).

Developing students' critical thinking skills remains a challenge for mathematics teachers in schools (Liu & Pásztor, 2022; Thi Nhat et al., 2018). Recently, Affandy et al. (2024) reported that students in Lampung Province, aged 12 to 14 years, face difficulties in solving mathematical problems due to insufficient critical thinking skills. Similarly, in Central Java Province, Dewi and Wijayanti (2022) found that students in Jepara District experienced challenges in solving math problems due to a lack of strong critical thinking skills. More generally, in the context of Indonesia, the 2015 Trends in International Mathematics and Science Study (TIMSS), an international study on trends in mathematics and science education, reported that Indonesian students' performance in mathematics was unsatisfactory (Mullis et al., 2016). Fourth-grade students ranked 44th out of 49 countries with an average score of 397. Additional data showing the low level of critical and creative thinking skills of Indonesian students was also found in the 2018 Program for International Student Assessment (PISA), where Indonesia’s performance ranked among the

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lowest 10. The average score of Indonesian students in mathematics was 379, indicating a decline compared to the 2015 PISA results.

Several studies have found that learning obstacles are one of the causes of low critical thinking skills in schools (Adharini & Herman, 2021; Sukri et al., 2023). Learning obstacles refer to the difficulties students encounter when learning mathematics, which hinder the achievement of learning objectives in an optimal way (Wijaya et al., 2019a). According to Brousseau (2002), learning obstacles in mathematics learning can be divided into three types: ontogenical learning obstacles, didactical learning obstacles, and epistemological learning obstacles. Ontogenical learning obstacles are obstacles caused by psychological aspects such as mental readiness, low motivation to learn, low self-confidence, and negative attitudes toward mathematics. Didactical learning obstacles are learning obstacles that arise due to the misalignment of teaching methods and inadequate presentation of teaching materials. The third type, epistemological learning obstacles, refers to difficulties caused by students' incomplete understanding of specific concepts. The last type is closely related to the cognitive schema possessed by the students.

To reduce learning obstacles, a framework is needed to help teachers understand how to plan and implement lessons that are appropriate for students' mental development levels, align with their cognitive schemas, and present material in ways that reduce learning obstacles. One framework that can guide the use of technology in the learning process is Didactical Design Research (DDR). DDR is a framework designed to create didactical designs based on the learning obstacles experienced by students (Suryadi, 2010). According to Suryadi (2010), several important dimensions need to be considered when designing didactical lessons, including the relationship between the teacher and students, which is called the pedagogical relationship, the relationship between students and the material, known as the didactical relationship, and the anticipatory relationship between the teacher and material, referred to as anticipatory didactical pedagogy. Through this framework, teachers can design didactical situations and predict students' responses and the corresponding anticipations.

Referring to the DDR framework, this study not only presents ways to teach effectively but also provides insights into how a well-structured lesson plan is developed by considering aspects of the material, the teacher, the students, and the relationships between these three components. This study aims to design a didactical framework for teaching volume to fifth-grade students in elementary school.

2. Method

This study uses the Didactical Design Research (DDR) approach. DDR is a phenomenological research method that aims to develop and design didactical frameworks, preceded by an analysis of students' learning obstacles (Suryadi, 2019). This didactical design research consists of three stages: (1) analysis of the didactical situation before the teaching process, which is in the form of a hypothetical didactical design, including the ADP (Didactical Analysis Framework), (2) meta-pedagogical analysis, and (3) retrospective analysis, which links the results of the analysis of the hypothetical didactical situation with the outcomes of meta pedagogical analysis (Suryadi, 2011). In this study, we only adopt the first stage, which is the analysis of the didactical situation and the design of the didactical framework. The designed didactical framework follows the framework of the Didactical Situations Theory (TDS) proposed by Brousseau (1997), which consists of action situations, formulation, and validation.

We observed the volume teaching process conducted by a teacher in a fifth-grade class at an elementary school in Gowa Regency, South Sulawesi. This class consists of 32 students aged between 10 and 12 years. Aspects observed include the use of teaching methods, the presentation of teaching materials, and the classroom dynamics during the teaching process. After observing the teaching process, we conducted interviews with the teacher and three students to explore the learning obstacles encountered during the lesson. Data on learning obstacles were also collected through written tests in the form of open-ended questions. The researchers then developed the lesson design based on the students' learning obstacles. All instruments used, including observation sheets, interview guidelines, and math competency tests, underwent strict validation testing.

3. Results

3.1. Learning obstacles

The learning obstacles experienced by students in geometry lessons on the topic of the volume of three-dimensional shapes include ontogenic obstacles, didactical obstacles, and epistemological obstacles. According to Brousseau (2002), didactical obstacles are obstacles caused by mental and psychological aspects. Didactical obstacles arise due to inappropriate teaching methods or because the teacher does not deliver the learning material effectively. Epistemological obstacles are obstacles that emerge from the approach to learning based on the concept itself. The learning obstacles encountered by the students are presented in the table below.

Table 1. Student Learning Obstacles Based on the Results of the Mathematics Test

Type Obstacle	Item				
	1	2	3	4	5
Ontogenic learning obstacle	10 (31.25%)	12 (37.50%)	8 (25%)	12 (37.50%)	9 (28.13%)
Didactic obstacle	17 (53.13%)	20 (62.50%)	21 (65.63%)	22 68.75%)	19 (59.38%)
Epistemological obstacle	19 (59.38%)	22 (68.75%)	18 (56.25%)	18 (56.25%)	27 (84.38%)

Table 1 shows the number of students who experienced various types of learning obstacles across five different items, with a total of 32 students involved in this study. Based on the data, there is a significant difference in the number of students who encountered each type of learning obstacle. For Ontogenic Learning Obstacle, the number of students facing difficulties ranged from 8 to 12 students per item, with percentages varying from 25% to 37.5%. Items 2 and 4 showed a higher number of students, with 12 students or 37.5%, indicating that individual or psychological factors played a larger role in these items.

Meanwhile, didactic obstacle was recorded as affecting most students, with a relatively high number of students in nearly all items, ranging from 53.13% in item 1 to 68.75% in item 4. Item 4, with 22 students (68.75%), showed that difficulties related to teaching methods or the delivery of the material were dominant factors for this item. Epistemological Obstacles also significantly impacted students, with the number of students experiencing difficulties ranging from 18 to 27 students, and a higher percentage in item 5, which reached 84.38%. This indicates that the understanding of the concept or knowledge in item 5 may have been more challenging for most students.

Overall, the didactic obstacle was the most commonly experienced by students, with the highest percentages recorded in almost every item. This shows that teaching factors and material delivery were the main obstacles encountered by students in their learning process. Additionally, although epistemological obstacles were also relatively high in some items, especially item 5, the

Didactic Obstacle remained the most dominant factor in the difficulties faced by students.

3.2. Didactic design of teaching volumes

The didactic design created is a lesson plan consisting of three meetings. This didactic design includes five main components: learning objectives, student activities, predicted responses, teacher didactic anticipations, and process evaluation. The ultimate learning goal formulated is for students to be able to analyze, present, and explain several problem-solving processes in detail and systematically related to the volume of cubes and rectangular prisms. Student activities are designed in such a way that students can construct their knowledge. For this purpose, student activities are organized based on the Didactic Situation Theory (TDS), which includes action situations, formulation, validation, and institutionalization. Each student activity is accompanied by a prediction of possible responses from students, and each predicted response is provided with teacher didactic anticipations and process evaluation. These predictions and didactic anticipations aim to keep the learning process under control. To achieve the formulated learning objectives, the learning trajectory that students will follow is divided into four meetings, as shown in the table below:

Table 2. Student Learning Trajectory

Pertemuan	Learning materials	student activities
1	Three dimensional shapes	Students engage in observational activities within the school environment to explore and identify basic geometric shapes, their characteristics, as well as the properties of cubes and rectangular prisms.
2	the volume of cube and cuboid	Students carry out experimental activities by inserting unit cubes into cube and cuboid containers to identify and understand the formula for the volume of cubes and cuboids.
3	Cube and cuboid volume problems	Students solve problems related to the volume of cubes and cuboids

The learning objectives for each session are designed to provide a deep understanding of solid shapes as well as the ability to analyze and solve problems related to them. In the first session, students are expected to explain basic solid shapes, analyze the parts of solid shapes, and identify the properties of cubes and rectangular prisms, while also solving problems related to the characteristics of these two shapes. In the second session, the focus shifts to the ability to explain the volume of cubes and rectangular prisms, as well as analyze and present the problem-solving process in a detailed and systematic manner concerning volume calculations. This objective is reiterated in the third session to ensure students' understanding of the volume of cubes and rectangular prisms, as well as their ability to present solutions to problems in an organized manner.

Table 3. Didactical Design for the First Session

Student Activities	Response Prediction	Didactic Anticipation (Teacher Support)	Process Assessment
Introduction: Students observe and identify simple geometric figures (cube, cuboid, pyramid, prism, cylinder, cone, sphere) starting from the sides, vertices, and edges.	Response 1: The cuboid has 16 ribs, 6 sides Response 2: The sides of the cube are square	Support 1: The teacher explains the properties of a cube: the number of edges and sides, the shape of the sides, corner points, diagonals, and space in the cube.	Can students identify the parts of a geometric shape?

First problem solving: Identify the properties of the cuboid!	Response 1: Cuboid has 16 edges, 6 sides Response 2: The sides of the cube are square	Support 1: The teacher explains the properties of a cube: the number of edges and sides, the shape of the sides, corner points, diagonals, and space in the cube.	Can students analyze the properties of cuboids?
Second problem solving Analyzing the problem: What are the differences and similarities between a cube and a cuboid?	Response 1: "The similarity is that it has 6 sides and the difference is that the sides of the cube are all square and the sides of the cuboid are rectangular or square" Response 2: "The sides of a cube are all the same size, but the sides have different sizes except the ones facing each other"	Support 1: The teacher explains that a cube is a cuboid that has a special characteristic, namely that it has the same size of sides/edges. Support 2: "What is the relationship between a cube, a cuboid, and a prism?"	Students can identify objects that include spatial shapes.
Third problem solving: Students identify relationships between cubes, cuboids, and prisms	Response 1: Drawing a triangular prism and can't find the relationship between cube, cuboid, prism	Support 1: The teacher presents triangular, rectangular, and pentagonal prisms and explains that these shapes are types of prisms. Support 2: "The prism base is not only triangular but can also be square and rectangular. Therefore, cubes and cuboids are prisms." Support 3: What about the tube, does it include a prism?	Students understand simple geometric figures, namely cubes, cuboids, pyramids, prisms, cones, and spheres.
Students identify whether the shape of the tube is a prism.	Response 1: "A tube is not a prism because its base and lid are not triangular, rectangular or other shaped"	Support 1: The teacher asks students to draw hexagons and nines.	Students understand the meaning of sides, edges, and vertices
Students draw and identify hexagons and octagons	Response 1: Students cannot draw conclusions	Support 1: The teacher explains that if you make a lot of n-facets. Then the image will approach a circle. Therefore, the tube is also a prism. Support 2: "If we draw sides 6,7,8 and so on, the resulting image will be close to a circle. The tube is an n-sided prism"	Students can identify the characteristics of cubes and cuboids.

The didactic design above was prepared based on Brousseau's didactic situation theory, which views learning as a dynamic interaction between students, teachers, and the concepts being studied. In this context, learning activities are designed so that students can go through three main stages, namely action situations, formulation situations, and validation situations. At the action situation stage, students are allowed to independently explore simple geometric shapes, such as cubes, cuboids, pyramids, and prisms. This stage is by Brousseau's (1997) view which states that students need to be in problematic situations that allow them to construct knowledge actively through exploratory actions.

Next, the formulation situation stage involves students in compiling, discussing, and formulating their understanding of the concepts being studied, such as the properties of cuboids and the relationships between cubes, cuboids, and prisms. In this stage, the teacher's didactic anticipation plays an important role as a form of scaffolding, namely gradual support to help students overcome the difficulties they face. Scaffolding theory, as explained by Wood, Bruner, and Ross (1976), emphasizes the importance of teacher intervention to guide students toward higher understanding through hints, trigger questions, or additional explanations. In this design, the teacher asks questions such as "Is a cylinder a prism?" or "What is the relationship between a cube and a prism?" to encourage students to think critically and develop their understanding.

The final stage, the validation situation, involves students presenting the results of their exploration and analysis, such as drawing hexagons and nineagons to understand that the cylinder can be viewed as an n-sided prism. The teacher's role is to ensure that the concepts constructed by students are following formal knowledge through additional explanations. This validation is also relevant to the scaffolding approach, where teachers help students to revise or confirm their understanding. By integrating Brousseau's theory of didactic situations and scaffolding, this design provides a systematic learning framework, where students not only learn actively but also receive didactical support appropriate to their needs to achieve an in-depth understanding of concepts.

Table 4. Didactical Design of The Second Meeting

Student Activities	Response Prediction	Didactic Anticipation (Teacher Support)	Process Assessment
Introduction: Students look for three numbers which, when multiplied, produce 27. This is done to remind them of previous material regarding number factors and cube root operations.	Response 1: obtained 3 x 3 x 3 and 9 x 3 x 1 by trial and error Response 2: Students use the cube root operation to obtain 3 x 3 x 3	Support 1: The teacher reminded the students regarding the cubic number, namely 27, as well as the cube operation to find the value the teacher meant Support 2: The teacher explains again regarding the factors of a number	Do students find the three numbers using root operations and know the factors of numbers?
Problem solving: Observe and analyze 3 mini aquariums provided by the teacher. If filled to the brim with water, which aquarium will contain the most and least water? The sizes of the aquariums provided (unknown to students) are: A: 4cm x 4cm x 4cm B: 8cm x 4cm x 2cm C: 4cm x 4cm x 3cm	Response 1: Choose aquarium A as the aquarium that will contain the most water because at a glance it looks bigger Response 2: Choose aquarium C as the aquarium that will hold the least amount of rice because at first glance it looks smaller.	Support 1: "If it were filled with rice, which aquarium would contain the most rice and which would contain the least?"	Do students observe the shape of the cube and its size well?
Students fill the aquarium with rice and then transfer the rice from the aquarium into a measuring cup.	Response 1: Aquariums A and B have the same amount of content, then Aquarium C has the least content.	Support 1: "Explain why that happened?" Support 2: "Why do aquariums A and B have the same contents?"	Will students predict/give answers based on the size of the aquarium?
Students measure the lengths on the sides of the aquarium	Response 1: Students find the size of the sides of the aquarium A: 4cm x 4cm x 4cm B: 8cm x 4cm x 2cm C: 4cm x 4cm x 3cm but cannot explain the relationship with the size	Support 2: Ask students to multiply the sizes of each side and compare one aquarium with another Support 2: Explain that the amount of rice in the aquarium is called volume.	Can students take measurements well?

<table>
<tr><td></td><td>of the aquarium contents

Response 2:
Students find the size of the sides of the aquarium and explain the product of each side:

A: 4cm x 4cm x 4cm
B: 8cm x 4cm x 2cm
C: 4cm x 4cm x 3cm</td><td>Support 3:
“How do you find the volume of a cube or cuboid?”

Support 4:
"To find out more about the volume, we will carry out further experiments."</td><td></td></tr>
<tr><td>Students put the unit cube in a mini aquarium.</td><td>Response 1:
Students record the number of unit cubes in each container</td><td>Support 1:
Present the following table and ask students to complete the data:
<table><tr><td>Container</td><td>l</td><td>w</td><td>h</td><td>Total</td></tr><tr><td>A</td><td>4</td><td>4</td><td>4</td><td>64</td></tr><tr><td>B</td><td></td><td></td><td></td><td></td></tr><tr><td>C</td><td></td><td></td><td></td><td></td></tr></table>
Support 2:
"What is the relationship between the number of unit cubes on each side and the total number of unit cubes?"

Support 3:
“What can you conclude?”</td><td>Can students find the relationship between the number of unit cubes on a side and the total of unit cubes?</td></tr>
<tr><td>Students identify the relationship between the number of unit cubes and the number of unit cubes in length, width, and height.</td><td>Response 1:
"The total unit cube is determined by the product of the number of each unit cube on the side of the container"

Response 2:
"To find the volume of a cube, the way to do it is to multiply the length of each side"</td><td>Support 1:
Explain that the number of unit cubes in a container is called volume and the way to find the volume in cubes and cuboids is to multiply the length, width, and height of the cube or cuboid or V = l x w x h</td><td>Do students know that the formula V = l x w x h is the same as V = s x s x s</td></tr>
</table>

In the second meeting, a didactic design was designed to help students understand the concept of volume of cubes and cuboids through an exploratory and experimental approach. The action situation stage can be seen from the activity of students who are asked to find three numbers which, when multiplied, produce 27. This activity aims to connect new material with students' initial knowledge about number factors and cube operations. The teacher provides support in the form of explanations regarding cubic numbers and reminders about cube operations, which is a form of scaffolding to facilitate students' thinking processes (Wood et al., 1976).

The formulation situation stage involves students in problem analysis through direct experiments, such as filling an aquarium with rice and measuring the contents using a measuring cup. This activity encourages students to observe the differences in volume of various aquarium sizes, involves them in the measurement process, and trains their ability to draw data-based conclusions. The teacher provides additional support in the form of questions, such as "Why do aquariums A and B have the same contents?" or "How do I calculate the volume of a cube or cuboid?", to guide students in relating length, width, height, and volume. With this approach, students are invited to build a deeper conceptual understanding.

In the validation situation stage, students carry out experiments by inserting unit cubes into a mini aquarium and recording the results in table form. The data obtained is used to find the relationship between length, width, height, and total volume. The teacher then explains that the volume of a cube or cuboid can be calculated using the formula V= l × w x h which can be simplified to V=s×s×s. This approach, which combines Brousseau's theory of didactic situations with scaffolding strategies, ensures students not only understand how to use formulas but are also able to explain the concept of volume logically through active exploration and validation of learning outcomes.

Table 5. Didactical Design of The Third Meeting

Student Activities	Response Prediction	Didactic Anticipation (Teacher Support)	Process Assessment
Introduction: Students analyze and state the characteristics of cubes and cuboids, as well as the formula for the volume of cubes and cuboids. This initial step is to ensure that students remember the characteristics of a cube/cube and the relationship between the volume and the sides of the cube/cuboid.	Response 1: State the number of edges and sides of a cube or cuboid Response 2: Explain that the lengths of the sides of a cube are the same	Support 1: The teacher reminds and directs that in terms of sides, the lengths of the sides of the cube are the same so to find the volume of the cube, the formula is $v=s^3$. So if the volume of a cube is known, then the concept of the cube root operation can be used to find the length of s.	Do students remember that the sides of a cube are the same?
First problem solving: Students analyze the cube/cuboid volume problem: A cube has a volume equal to the volume of a cuboid. If the cuboid is 12cm long, 8cm wide, and 18cm high. What is the length of the edge of the cube?	Response 1: Identify and write down information on the problem, starting from things that are known and things that are asked. Response 2: Students do not write down the information V.cube=V.Cuboid	Support 1: The teacher provides scaffolding and guides students to analyze the information contained in the problem.	Are students able to identify the information in the question?
Students find the volume of a cube by multiplying the length, width, and height. V= 12 x 8 x 18	Response 1: Students are unable to draw a relationship between the results of calculating the volume of the cuboid and the length of the edge of the cube	Support 1: The teacher provides scaffolding by reminding the relationship that V.cube=V.cuboid Support 2: The teacher guides students to remember the cube volume formula	Are students able to determine problem-solving strategies and explain problem-solving steps?
Students write the relationship between the volume of the cuboid and the volume of the cube. V.cube=V.cuboid = 1.728	Response 1: Students do not know the relationship between the volume of the cuboid and the length of the side of the cube they are looking for	Support 1: The teacher provides scaffolding and helps students to find the relationship between the three V.cuboid=V.cuboid = 1.728 = S^3	Are students able to explain the relationship between the volume of the cube, the volume of the cuboid, and the edges of the cuboid that will be searched for?
Students find the length of the cube using the previously found value of the volume of the cube and prove the answer. v. cube = $\sqrt[3]{1728}$= 12	Response 1: Students provide conclusions and prove their answers again by calculating V.cube based on the length of the edge they have obtained V = 12^3 = 1728.	Support 1: The teacher directs students to prove the correctness of the answers that have been found.	Are students able to prove the correctness of the answers obtained?

The didactic design for the third meeting is designed to help students understand and apply the concept of the volume of cubes and cuboids in various real problem contexts. This design is based on Brousseau's theory of didactic situations and uses a scaffolding approach to ensure students can go through the stages of action, formulation, and validation situations gradually. At the action situation stage, students are invited to analyze and recall the characteristics of cubes and cuboids, including the formula for the volume of these two geometric figures. This activity aims to connect previous knowledge with new material. The teacher provides support in the form of an explanation of the properties of a cube, such as equal side lengths, and reminds that the formula for the volume of a cube is Volume= s^3 which can also be used to find the length of the sides using the cube root operation.

At the formulation situation stage, students are given problems to apply the concept of volume in a more challenging context. In the first problem, students are asked to calculate the edge lengths of a cube with the same volume as the cuboid. The teacher helps students identify important information in the problem, such as the relationship between the formula for the volume of a cube and the volume of a cuboid, and guides them to find the side lengths of a cube from a known volume. This process helps students understand the relationship between the volume of a cuboid, the volume of a cube, and the length of the sides of a cube so that they can develop steps to solve problems in a logical and structured way.

4. Discussion

The results of this research reveal that there is a complex interaction between three types of learning obstacles faced by students, namely ontogenic, didactic, and epistemological obstacles. Although each of these types of obstacles focuses on different aspects of the learning process, they influence each other and can help each other. Ontogenic obstacles relate to student's individual characteristics, such as cognitive development or learning readiness, which influence their ability to understand material. In contrast, didactic obstacles are more related to the teaching methods and strategies used by teachers, which can influence how effectively students understand the material being taught. Epistemological obstacles, on the other hand, occur when students have difficulty understanding or accessing certain concepts, especially if the material is considered too abstract or difficult to understand.

The findings in this research show that didactic obstacles are often the main factor that worsens epistemological and ontogenic obstacles. This is in line with findings in previous research, as expressed by Artigue (2009), which stated that obstacles in the teaching process, such as a lack of clarity in explanations or choosing methods that do not suit students' needs, can worsen their understanding of the material. Other research by Schoenfeld (2016) also emphasizes the importance of paying attention to individual student differences in teaching, because mismatching methods with students' abilities or backgrounds can lead to greater difficulties in learning. If teaching is not adapted to students' level of readiness or cognitive abilities, then ontogenic and epistemological obstacles become more difficult to overcome (Santana & Nery, 2022).

Moreover, the relationship between these three obstacles suggests that ineffectiveness in teaching may exacerbate students' epistemological difficulties, given that they do not receive clear enough explanations or in-depth instruction about the material. Research by Carpenter and Lehrer (1999) supports this by showing that the inability to relate new knowledge to students' previous experiences can lead to difficulties in understanding new concepts. Furthermore, the study by Clements and Sarama (2004) also emphasizes that ambiguity in teaching and a lack of connection between theory and practice can exacerbate learning obstacles, especially in the context of mathematics and science learning. Overall, the findings from this research highlight the importance of a more holistic and contextual approach to overcoming obstacles to learning. Such an approach should involve improvements in teaching strategies as well as taking into account factors in individual student development and their understanding of the material being taught.

The didactic design framework in the form of TDS initiated by (Brousseau, 1997) emphasizes the importance of designing didactic situations that allow students to be involved in the process of discovery and construction of knowledge. In TDS, students not only solve mathematical problems but also build understanding through interaction with assignments provided by the teacher (Dani & Badarudin, 2019; Wijaya et al., 2019b). As explained by Brousseau (1997), effective learning occurs when students can develop their ideas through challenging but accessible situations, which further strengthens their understanding.

Meanwhile, Learning Trajectory (LT) provides a path map that helps teachers design gradual learning according to students' cognitive development (Clements & Sarama, 2004). From another perspective, the idea of the Zone of Proximal Development (ZPD), a concept developed by Vygotzky (1978), refers to the range of abilities that students can achieve with support from other people, such as teachers or classmates. ZPD covers the area between what a student can do independently and what can be achieved with help. ZPD's relationship to TDS and Discovery Learning is critical because TDS focuses on designing situations that bring students into their ZPD, enabling them to learn through higher challenges than they could achieve on their own. Vygotsky (1978) emphasized that in ZPD, students can develop further with the right support, which can be in the form of scaffolding.

Scaffolding, a concept introduced by Wood et al., (1976), refers to the assistance provided by teachers to help students through difficult tasks. Wood et al. (1976) revealed that scaffolding helps students complete tasks that they cannot do themselves, but are still within the range of their abilities with help. Scaffolding can take the form of open-ended questions, feedback, or providing hints that allow students to think more critically and reach better solutions. In mathematics learning, scaffolding allows students to overcome obstacles in understanding concepts, while keeping them within their ZPD (Wood, Bruner, & Ross, 1976).

An approach that combines TDS, Learning Trajectory, ZPD, and Scaffolding, can produce more meaningful learning, which is appropriate to students' developmental needs. As stated by Steffe and Kieren (1994), when teachers design learning based on TDS and LT, they provide students with a clear path to more complex understanding, while facilitating learning with the necessary support. This encourages students to continue to develop, both cognitively and in their mathematical skills.

This research adopts a Didactical Design Research (DDR) approach which emphasizes the development of didactical designs for volume learning, to improve students' critical thinking abilities. The focus of this research is limited to the first stage of DDR, namely the didactic situation analysis carried out before learning. For future research, we suggest that researchers proceed to the metapedadidactic analysis and retrospective analysis stages, to evaluate the extent to which this didactic design can be adapted to the real conditions of students in the field.

5. Conclusion

The conclusions of this research reveal the existence of a complex interaction between three types of learning obstacles experienced by students, namely ontogenic, didactic, and epistemological obstacles. Although each obstacle focuses on a different aspect, such as individual development, teaching methods, and difficulties in understanding abstract concepts, all three interact and exacerbate each other. Research findings show that didactic obstacles, such as the mismatch of teaching methods with students' needs, are the main factors that exacerbate ontogenic and epistemological obstacles. This emphasizes the importance of teaching approaches that are adapted to students' abilities and cognitive development. In addition, theories such as the Didactical Situation Theory (TDS), Learning Trajectory (LT), and Zone of Proximal Development (ZPD) provide insight into how learning can be designed to overcome these obstacles, using scaffolding as support for student development. Therefore, a more comprehensive and contextual teaching approach is needed to help students understand the material and improve their critical thinking skills. This research also recommends that further research proceed to the metapedadidactic and retrospective analysis stages to evaluate the extent to which the didactic design developed can be applied in real learning conditions.

6. Declarations

6.1. Funding

Not applicable

6.2. Conflicts of Interest

The author declares that there is no conflict of interest regarding the publication of this manuscript. Apart from that, the author has also paid attention to aspects of research in the form of ethics, falsification and plagiarism.

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